STUDY OF A C-11 INSTRUMENT FLIGHT TRAINER AS A VARIABLE STABILITY MODEL AND SIMULATOR FOR STUDIES IN FLIGHT DYNAMICS

JOHN ARNOLD JOHNSON

[redacted]

STUDY OF A C-11 INSTRUMENT FLIGHT TRAINER AS A VARIABLE STABILITY MODEL AND SIMULATOR FOR STUDIES IN FLIGHT DYNAMICS

by

John Arnold Johnson
Lieutenant, United States Naval Reserve

Submitted in partial fulfillment of the
requirements for the degree of

MASTER OF SCIENCE IN AERONAUTICAL ENGINEERING

from the

NAVAL POSTGRADUATE SCHOOL
December 1967

1967

JOHNSON, J.

ABSTRACT

A study has been made of a C-11 Instrument Flight Trainer to investigate its capabilities as a variable stability model and simulator. The cockpit, with its controls and instruments, and the power supplies were found to be suitable for this purpose. All other components were not. The existing computers have been simplified to handle only linearized approximations of the aircraft stability equations of motion. An analog computer will have to be designed to solve the equations of motion in higher order differential form. In addition, systems will be required to provide simulated cockpit motion and visual terrain reference.

Thesis of LT John A. Johnson, titled "Study of C-11 Instrument Flight Trainer as a Variable Stability Model and Simulator for Studies in Flight Dynamics"

THESIS ERRATA

PAGE	LINE	CHANGE
5	Item 1	Insert "x", "y", & "z" in appropriate spaces
7	1, 2, 3, & 7	"ft/sec" to "ft/sec^2"
21	3 of text	Insert "θ" after "then" (end of line)
29	5th	"wind axes system--- to "wind axes" system.
47	last paragraph line 1	concern- to concerned
48	last paragraph line 4	"such as been" to "such has been"
49	1st	"cue and" to "cue, an"

TABLE OF CONTENTS

CHAPTER | Page

I. INTRODUCTION 11

II. DEVELOPMENT OF THE EQUATIONS OF MOTION 13

Assumptions 13

The Effects of Gravity Forces 20

Expansion of the Aerodynamic Forces and Moments 29

Solution by Analog Computer 34

III. DESCRIPTION OF THE C-11 TRAINER 38

Methods of Computation in the Trainer 39

Longitudinal Acceleration 39

Lateral Acceleration 40

Normal Acceleration 41

Angle of Attack 41

Angle of Bank 42

Pitching Rate 42

Angle of Yaw 43

Flight Path Elevation Angle 43

Vertical Velocity 44

Altitude 44

Mach Number 44

Dynamic Pressure 45

IV. EVALUATION OF THE TRAINER 46

V. CONCLUSIONS 51

REFERENCES 52

APPENDIX I.- SYMBOLS USED IN C-11 INSTRUMENT FLIGHT TRAINER SCHEMATICS 53

APPENDIX II. - INSTRUMENT FLIGHT TRAINER COMPUTATION SCHEMATICS. 55

LIST OF FIGURES

FIGURE PAGE

1. X, Y, Z, Axes Fixed in Space and Invariant with Time... 14

2. Rotation [illegible] About the [illegible] Axis.............................. 21

3. Rotation Φ_0 About the x Axis.............................. 21

4. Rotation ψ About the z Axis.............................. 21

5. Rotation θ About the y_1 Axis.............................. 22

6. Rotation φ About the x_2 Axis.............................. 22

7. Analog Longitudinal Motion.................................. 35

8. Analog Vertical Motion...................................... 35

9. Analog Yawing Motion.. 36

10. Analog Lateral Motion....................................... 36

11. Analog Pitching Motion...................................... 37

12. Analog Rolling Motion....................................... 37

13. Aircraft Variable Stability Model and Simulator........... 50

TABLE OF SYMBOLS

a_x Acceleration along the x axis, ft/sec^2

a_y Acceleration along the y axis, ft/sec^2

a_z Acceleration along the z axis, ft/sec^2

F_x Sum of forces in the x direction, lbs

F_y Sum of forces in the y direction, lbs

F_z Sum of forces in the z direction, lbs

g Acceleration of gravity, ft/sec^2

$\bar{h}$ Angular momentum vector, lb ft sec

h_x Angular momentum component about the x axis, lb ft sec

h_y Angular momentum component about the y axis, lb ft sec

h_z Angular momentum component about the z axis, lb ft sec

I_x Moment of inertia about the x axis, lb ft sec

I_y Moment of inertia about the y axis, lb ft sec

I_z Moment of inertia about the z axis, lb ft sec

J_{xy} Product of inertia in the x, y plane, lb ft sec

J_{xz} Product of inertia in the x, z plane, lb ft sec

J_{yz} Product of inertia in the y, z plane, lb ft sec

L Rolling moment about the x axis, lb ft

L_0 Initial rolling moment, lb ft

L_p Rolling moment derivative due to angular roll velocity, $\frac{\text{lf ft sec}}{\text{rad}}$

L_r Rolling moment derivative due to yawing velocity, $\frac{\text{lb ft sec}}{\text{rad}}$

L_v Rolling moment derivative due to lateral velocity, lb sec

$L_{\delta a}$ Rolling moment derivative due to aileron deflection, $\frac{\text{lb ft}}{\text{rad}}$

$L_{\delta r}$ Rolling moment derivative due to rudder deflection, $\frac{\text{lb ft}}{\text{rad}}$

M	Pitching moment about the y axis, lb ft
M_o	Initial Pitching moment, lb ft
M_q	Pitching moment derivative due to pitch rate, $\frac{\text{lb ft sec}}{\text{rad}}$
M_u	Pitching moment derivative due to forward velocity, lb sec
M_w	Pitching moment derivative due to vertical velocity, lb sec
$M_{\delta e}$	Pitching moment derivative due to elevator deflection, $\frac{\text{lb ft}}{\text{rad}}$
m	Mass of the aircraft, $\frac{\text{lb sec}^2}{\text{ft}}$
N	Yawing moment about the z axis, lb ft
N_o	Initial yawing moment, lb ft
N_p	Yawing moment derivative due to roll, $\frac{\text{lb ft sec}}{\text{rad}}$
N_r	Yawing moment derivative due to yaw, $\frac{\text{lb ft sec}}{\text{rad}}$
N_v	Yawing moment derivative due to lateral velocity, lb sec
$N_{\delta a}$	Yawing moment derivative due to aileron deflection, $\frac{\text{lb ft}}{\text{rad}}$
$N_{\delta r}$	Yawing moment derivative due to rudder deflection, $\frac{\text{lb ft}}{\text{rad}}$
p	Angular roll perturbation velocity, rad/sec
P_o	Initial rolling velocity, rad/sec
P	Total rolling velocity, rad/sec
q	Angular pitch perturbation velocity, rad/sec
Q_o	Initial angular pitching velocity, rad/sec
Q	Total angular pitching velocity, rad/sec
r	Angular yaw perturbation velocity, rad/sec
R_o	Initial yawing velocity, rad/sec
R	Total yawing velocity, rad/sec
U_o	Initial steady state forward velocity, ft/sec

u Forward perturbation velocity, ft/sec

U Total forward velocity, ft/sec

V_o Initial steady state lateral velocity, ft/sec

$\overline{V}_T$ Total aircraft velocity vector, ft/sec

V Total lateral velocity, ft/sec

v Lateral perturbation velocity, ft/sec

W_o Initial steady state vertical velocity, ft/sec

W Total vertical velocity, ft/sec

w Vertical perturbation velocity, ft/sec

w_t Weight of the aircraft, lbs

$\mathcal{X}_q$ Longitudinal force derivative due to pitching rate, $\frac{\text{lb sec}}{\text{rad}}$

$\mathcal{X}_u$ Longitudinal force derivative due to forward velocity, $\frac{\text{lb sec}}{\text{ft}}$

$\mathcal{X}_w$ Longitudinal force derivative due to vertical velocity, $\frac{\text{lb sec}}{\text{ft}}$

$\mathcal{X}_{\delta e}$ Longitudinal force derivative due to elevator deflection, lb/rad

$\mathcal{Y}_p$ Lateral force derivative due to roll rate, $\frac{\text{lb sec}}{\text{rad}}$

$\mathcal{Y}_r$ Lateral force derivative due to yaw rate, $\frac{\text{lb sec}}{\text{rad}}$

$\mathcal{Y}_v$ Lateral force derivative due to lateral velocity, $\frac{\text{lb sec}}{\text{ft}}$

$\mathcal{Y}_{\delta a}$ Lateral force derivative due to aileron deflection, lb/rad

$\mathcal{Y}_{\delta r}$ Lateral force derivative due to rudder deflection, lb/rad

$\mathcal{Z}_q$ Vertical force derivative due to pitching rate, $\frac{\text{lb sec}}{\text{rad}}$

$\mathcal{Z}_u$ Vertical force derivative due to forward velocity, $\frac{\text{lb sec}}{\text{ft}}$

$\mathcal{Z}_w$ Vertical force derivative due to vertical velocity, $\frac{\text{lb sec}}{\text{ft}}$

$\mathcal{Z}_{\delta e}$ Vertical force derivative due to elevator deflection, lb/rad

δ_a Aileron deflection, rad

δ_e Elevator deflection, rad

δ_r Rudder deflection, rad

θ_o Initial steady state Euler angle in the x, z plane, rad

θ Euler perturbation angle in the x, z plane, rad

φ_o Initial steady state Euler angle in the y, z plane, rad

φ Euler perturbation angle in the y, z plane, rad

ψ_o Initial steady state Euler angle in the x, y plane, rad

ψ Euler perturbation angle in the x, y plane, rad

λ Assumed root of the stability quartic equation

$\overline{\omega}_T$ Angular velocity vector of the aircraft, rad/sec

CHAPTER I

INTRODUCTION

A variable stability model of an aircraft is a laboratory device which simulates the motion and response to control variation of an aircraft in flight. The aircraft is described by parameters such as mass, moments of inertia, geometry, propulsive power, etc. These in turn are expressed in the form of coefficients and derivatives, the values of which may be arbitrarily assigned and varied making the design of the simulated aircraft variable. When such values are magnitudes of voltage established by potentiometers, they can be introduced to an analog computer which represents, in electronic circuits, the Newtonian equations of motion. The analog computer solves these equations of motion and produces electrical signals representing the displacement, velocities and accelerations that the simulated aircraft would experience if it were in actual flight. These signals can be plotted on graphs, be displayed as instrument readings on a simulated instrument panel, provide forces on simulated cockpit controls, or drive a mockup model of the aircraft if it has the corresponding degrees of freedom.

If the computer inputs which represent the aerodynamic and propulsion controls are generated by use of manual control devices in a cockpit mockup, a pilot can, in a sense, fly the stability model as if it were an actual aircraft.

In this way an aircraft does not have to be built and flown in order to evaluate its flight and handling characteristics. Estimated changes in design can be made on the spot and results immediately compared. More important, the margins of stability and

and control can be investigated in detail without exposing personnel and equipment to the risk of catastrophic failure.

An instrument flight trainer, which is normally used for the training and maintenance of proficiency of pilots in instrument flight and navigation procedures, consists of a cockpit mockup with instruments and controls as well as navigation and communications equipment. It responds to the pilot's use of the controls by way of instrument readings, control forces and, in some types, motion of the mockup within certain limited degrees of freedom simulating actual flight.

These are the same characteristics as those ascribed to the stability model. The exception is that the trainer is based on the response of one aircraft design whose parameters and derivatives are fixed constants rather than any that can be variably assigned. If the resistances which establish these fixed parameters can be replaced by potentiometers, and if the analog circuits of the trainer are fully compatible with the equations of motion, the trainer can be converted into a variable stability model and simulator.

The subject trainer is a Link Aviation type C-11 Instrument Flight Trainer which has been acquired by the Department of Aeronautics of the Naval Postgraduate School. The following study has been made to determine the suitability of this trainer for conversion into a variable stability model and simulator.

CHAPTER II

DEVELOPMENT OF THE EQUATIONS OF MOTION [1]

The basic function of the flight simulator is to solve the simulated equations of motion of the aircraft by means of analog circuitry. This results in instrument readings indicative of position, attitude, and rates of change. Such results must be dimensional and in real time if a subject pilot is to properly respond to them. To provide such results, the original equations of motion must be fully dimensional and in real time. Since the classical analysis of aircraft stability makes use of nondimensional equations of motion in aerodynamic time, it is necessary to develop more suitable equations for a flight simulator.

Assumptions

The following assumptions can be made:

a. The aircraft is assumed to be a rigid body.

b. Earth-air coordinates are assumed to be fixed in space with no wind.

c. The mass of the aircraft remains constant.

d. The x,z plane is a plane of symmetry.

d. Disturbances from steady flight are small. Products and higher orders of velocity increments are negligible.

Starting with Newton's Second Law:

Force equals the time rate of change of momentum.

$$\sum F_x = \frac{d}{dt}(mu)$$

$$\sum F_y = \frac{d}{dt}(mv)$$

$$\sum F_z = \frac{d}{dt}(mw)$$

Torque equals the time rate of change of moment of momentum.

$$\sum L = \frac{dh_x}{dt}$$

$$\sum M = \frac{dh_y}{dt}$$

$$\sum N = \frac{dh_z}{dt}$$

The coordinate system and sign convention adopted is shown in Figure 1.

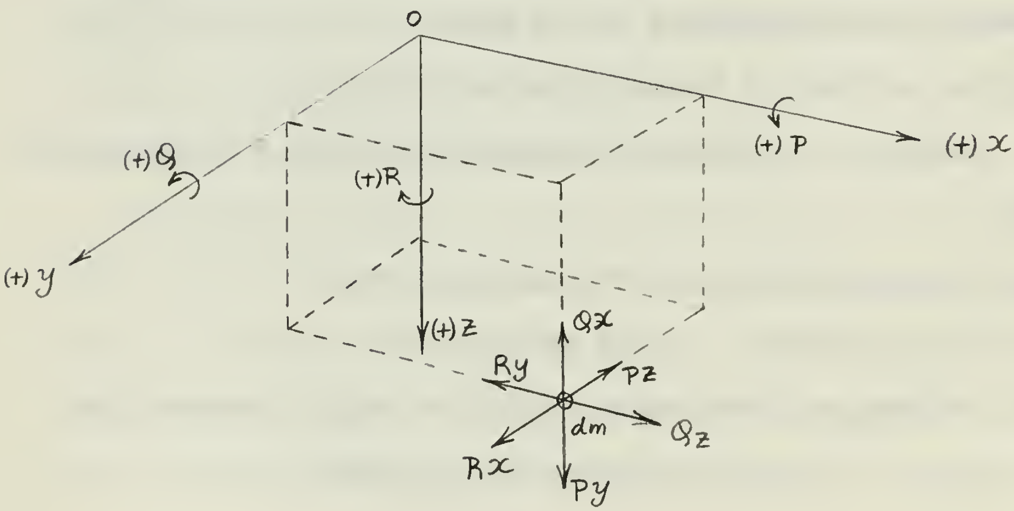


Figure 1

Axes Fixed in Space and Invariant with Time

The components of Angular momentum are:

$$dh_x = \left[Pz(z) + Py(y) - Qx(y) - Rx(z)\right] dm$$

$$= P(x^2 + z^2)\,dm - Q\,xy\,dm - R\,xz\,dm$$

$$dh_y = \left[Qx(x) - Py(x) + Qz(z) - Ry(z)\right] dm$$

$$= -P\,xy\,dm + Q(x^2 + z^2) - R\,yz\,dm$$

$$dh_z = \left[-Pz(x) + Rx(x) + Ry(y) - Qz(y)\right] dm$$

$$= -P\,xz\,dm - Q\,yz\,dm + R(x^2 + y^2)\,dm$$

Integrating:

$$h_x = P I_x - Q J_{xy} - R J_{xz}$$

$$h_y = -P J_{xy} + Q I_y - R J_{yz}$$

$$h_z = -P J_{xz} - Q J_{yz} + R I_z$$

The sum of forces becomes:

$$\sum F_x = m\frac{dU}{dt} = m a_x$$

$$\sum F_y = m\frac{dV}{dt} = m a_y$$

$$\sum F_z = m\frac{dW}{dt} = m a_z$$

The sum of moments becomes:

$$\sum L = \frac{dh_x}{dt} = \dot{P} I_x + P \dot{I}_x - \dot{Q} J_{xy} - Q \dot{J}_{xy} - \dot{R} J_{xz} - R \dot{J}_{xz}$$

$$\sum M = \frac{dh_y}{dt} = -\dot{P} J_{xy} - P \dot{J}_{xy} + \dot{Q} I_y + Q \dot{I}_y - \dot{R} J_{yz} - R \dot{J}_{yz}$$

$$\sum N = \frac{dh_z}{dt} = -\dot{P} J_{xz} - P \dot{J}_{xz} - \dot{Q} J_{yz} - Q \dot{J}_{yz} + \dot{R} I_z + R \dot{I}_z$$

Note the presence of time rate of change of moment of inertia.

To relate the equations of motion to Eulerian axes, let axes x_1, y_1, and z_1 be orthogonal axes fixed in the aircraft with origin at the center of gravity. The aircraft can then have linear and angular velocities and accelerations, but no displacement. The velocities and accelerations will be those which would be measured by the instruments in the simulated aircraft, oriented with respect to the three axes, while the moments and products of inertia remain unchanged with aircraft motion.

Let such a set of axes have an angular velocity $\bar{\omega}_T$ having components P, Q, and R about x_1, y_1, and z_1 axes and a linear velocity $\bar{V}_T$ having components U, V, and W along the same axes.

The angular and linear velocities are related in that $V_T = \bar{\omega}_T \times \bar{\rho}$ where $\bar{\rho}$ is the distance between the set of axes moving with the aircraft and the point, or center, fixed in space.

If the angular and linear velocities of the aircraft axes are held constant without angular and linear accelerations, the aircraft will still experience an acceleration. Often referred to as "centripetal" acceleration, it is the result of the cross product $\bar{\omega}_T \times (\bar{\omega}_T \times \bar{\rho})$. Since $\bar{V}_T = \bar{\omega}_T \times \bar{\rho}$, the absolute acceleration of the simulated aircraft with constant velocity is $\bar{a} = \bar{\omega}_T \times \bar{V}_T$.

If $\bar{V}_T$ is allowed to increase in the absence of angular velocity, meaning straight flight, then $\bar{a} = \frac{d\bar{V}_T}{dt}$.

For the case of both linear and angular velocities with linear acceleration:

$$\bar{a} = \frac{d\bar{V}_T}{dt} + \bar{\omega}_T \times \bar{V}_T$$

where:

$$\bar{\omega}_T \times \bar{V}_T = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ U & V & W \end{vmatrix} = (QW - RV)\hat{i} + (RU - PW)\hat{j} + (PV - QU)\hat{k}$$

and:

$$\frac{d\bar{V}_T}{dt} = \frac{d}{dt}(U\hat{i} + V\hat{j} + Wk) = \dot{U}\hat{i} + \dot{V}\hat{j} + \dot{W}\hat{k}$$

The components of $\bar{a}$ will be:

$$a_x = \dot{U} + QW - RV$$
$$a_y = \dot{V} - PW + RU$$
$$a_z = \dot{W} + PV - QU$$

The equations for the moment of momentum about the moving x_1, y_1, z_1, axes can be expressed in the same form as for the fixed axes, where now the moments and products of inertia will be for the aircraft axes.

$$h_{x_1} = PI_x - QJ_{xy} - RJ_{xz}$$

$$h_{y_1} = -PJ_{xy} + QI_y - RJ_{yz}$$

$$h_{z_1} = -PJ_{xz} - QJ_{yz} + RI_z$$

where: $\bar{h} = \bar{h}(x_1, y_1, z_1) = h_{x_1}\hat{i} + h_{y_1}\hat{j} + h_{z_1}\hat{k}$

Since $\bar{h}$ is a vector quantity having components h_{x_1}, h_{y_1}, and h_{z_1}, it can be treated in the same manner as the velocity vector $\bar{V}_T$, which, when rotated at an angular velocity, $\bar{\omega}_T$, gave a linear acceleration. However, when the moment of momentum, or angular momentum, vector is rotated at an angular velocity the result is an angular acceleration.

Thus: $$\frac{d\bar{H}}{dt} = \frac{d\bar{h}}{dt} + \bar{\omega}_T \times \bar{h}$$

where $\bar{H}$ is the absolute moment of momentum about a point fixed in space, and $\bar{h}$ is the moment of momentum about the moving aircraft axes.

In other words a roll must be produced by a torque; but if the rolling aircraft is also yawed while it rolls, additional torques or moments must be applied in order to maintain equilibrium.

Expanding the cross product $\bar{\omega}_T \times \bar{h}$:

$$\bar{\omega}_T \times \bar{h} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ P & Q & R \\ h_{x_1} & h_{y_1} & h_{z_1} \end{vmatrix}$$

$$= (Qh_{z_1} - Rh_{y_1})\hat{i} + (Rh_{x_1} - Ph_{z_1})\hat{j} + (Ph_{y_1} - Qh_{x_1})\hat{k}$$

Expressing the rate of change of the absolute moment of momentum in component form and recalling that the moments and products of inertia are invariant with time:

$$\frac{dH_x}{dt} = \dot{P}I_x - \dot{Q}J_{xy} - \dot{R}J_{xz} + QR(I_z - I_y) - PQJ_{xz} - Q^2J_{yz} + R^2J_{yz} + PRJ_{xy}$$

$$\frac{dH_y}{dt} = -\dot{P}J_{xy} + \dot{Q}I_y - \dot{R}J_{yz} + PR(I_x - I_z) - QRJ_{xy} - R^2J_{xz} + P^2J_{xz} + PQJ_{yz}$$

$$\frac{dH_z}{dt} = -\dot{P}J_{xz} - \dot{Q}J_{yz} + \dot{R}I_z + PQ(I_y - I_x) - PRJ_{yz} - P^2J_{xy} + Q^2J_{xy} + QRJ_{xz}$$

So far the x_1, y_1, z_1 axis system has been arbitrary as to orientation in the aircraft. Now with the assumption that the x_1, z_1 plane is a plane of symmetry, which is true for most aircraft, the size of the equations are greatly reduced. This is because there is both a positive and negative y_1 of equal value for each x_1 and z_1 value.

Thus: $\int yz\,dm = \int xy\,dm = 0$ or $J_{yz} = J_{xy} = 0$

The expanded and simplified equations of motion with respect to the Eulerian axes are then:

$$\sum F_x = m\left[\dot{U} + QW - RV\right]$$

$$\sum F_y = m\left[\dot{V} - PW + RU\right]$$

$$\sum F_z = m\left[\dot{W} + PV - QU\right]$$

$$\sum L = \dot{P} I_x - \dot{R} J_{xz} + QR(I_z - I_y) - PQ J_{xz}$$

$$\sum M = \dot{Q} I_y + PR(I_x - I_z) - R^2 J_{xz} + P^2 J_{xz}$$

$$\sum N = \dot{R} I_z - \dot{P} J_{xz} + PQ(I_y - I_x) + QR J_{xz}$$

The preceding forces and moments are those due to gravity, aerodynamic and thrust forces. If equilibrium flight is defined as unaccelerated flight where $\frac{d\bar{V}}{dt} = 0$ and $\bar{\omega} = 0$, and steady flight, as that for which $\dot{U}$, $\dot{V}$, $\dot{W}$, $\dot{P}$, $\dot{Q}$, $\dot{R} = 0$, then a flight at a constant rate of turn is steady flight which originated

from equilibrium flight.

Disturbed motion can be considered to be a change from steady flight, and the Eulerian axes under such conditions are considered to be disturbed axes. The forces can now be considered to consist of steady flight forces plus increments of force due to disturbances.

The Effect of Gravity Force

Since the force of gravity continually acts toward the center of the earth, its components act along the Eulerian axes which are not fixed in space. External moments are not affected because the gravity force acts through the center of gravity of the aircraft. However, the external forces in the three Eulerian directions are affected and must be determined.

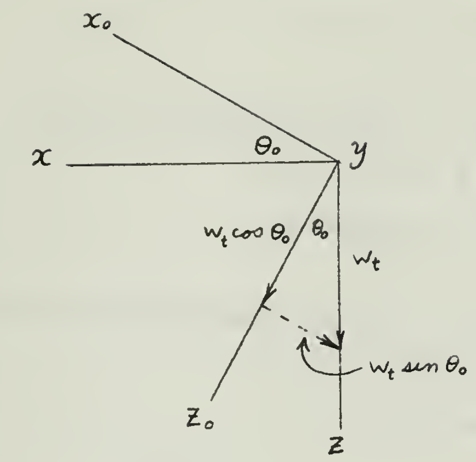
It is first necessary to find the components of the weight vector along the steady flight Eulerian axes, x_0 , y_0 , and z_0 ; then they are to be transferred to "disturbed" Eulerian axes which are rotated through Eulerian angles φ , θ , and ψ about the x , y , and z axes. Consider the x_0 , y_0 , and z_0 axes displaced from the horizontal and vertical, relative to the earth, by angles θ_0 , φ_0 , and ψ_0 . Rotation about the z axis through ψ_0 does not give a component of the weight vector about any other axis.

From Figures 2 and 3 it can be seen that:

$$x_0 = - w_t \sin \theta_0$$

$$y_0 = w_t \cos \theta_0 \sin \varphi_0$$

$$z_0 = w_t \cos \theta_0 \cos \varphi_0$$



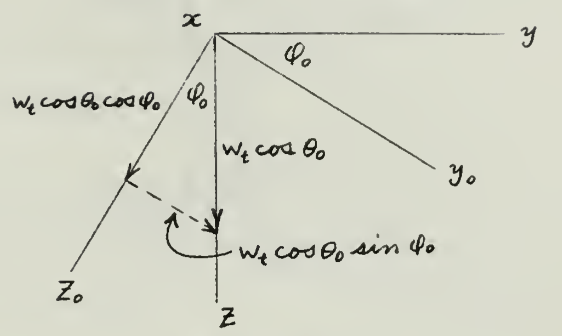


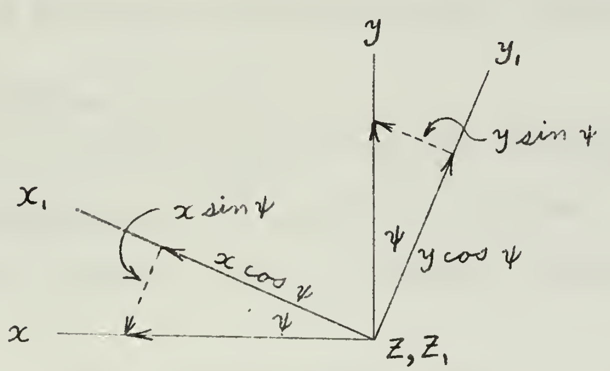
Figure 2

Rotation θ_0 about the y Axis

Figure 3

Rotation φ_0 about the x Axis

These are the weight components acting along the steady or initial flight axes. To develop the components along the disturbed axes three steps are taken: First, rotation ψ about the z axis, then θ about the rotated y axis, and then φ about the rotated x axis. These steps are shown in Figures 4, 5, and 6 respectively.



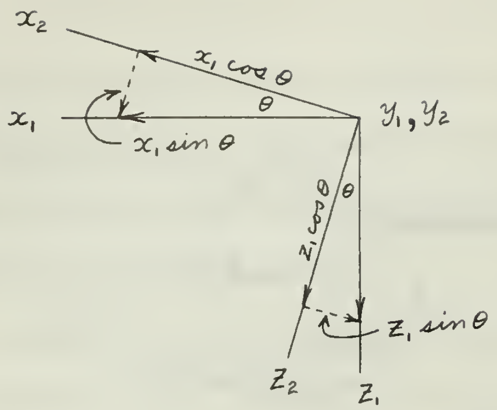
$$x_1 = x\cos\psi + y\sin\psi$$

$$y_1 = y\cos\psi + x\sin\psi$$

$$z_1 = z$$

Figure 4

Rotation ψ about the z Axis



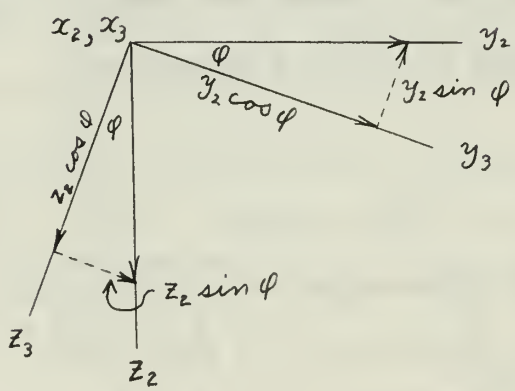
$$x_2 = x_1 \cos\theta - z_1 \sin\theta$$

$$y_2 = y_1$$

$$z_2 = z_1 \cos\theta + x_1 \sin\theta$$

Figure 5

Rotation θ about the y_1 Axis



$$x_3 = x_2$$

$$y_3 = y_2 \cos\varphi + z_2 \sin\varphi$$

$$z_3 = z_2 \cos\varphi - y_2 \sin\varphi$$

Figure 6

Rotation φ about the x_2 Axis

Combining the three sets of equations:

$$x_3 = x \cos\theta \cos\psi + y \cos\theta \sin\psi - z \sin\theta$$

$$\begin{aligned} y_3 = {} & x(\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi) \\ & + y(\cos\psi \cos\phi + \sin\psi \sin\theta \sin\phi) \\ & + z(\cos\theta \sin\phi) \end{aligned}$$

$$\begin{aligned} z_3 = {} & x(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) \\ & + y(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) \\ & + z(\cos\theta \cos\phi) \end{aligned}$$

Substituting x_o , y_o , and z_o for x , y , and z :

$$x_3 = (-W_t \sin\theta_o)\cos\theta\cos\psi + (W_t \cos\theta_o \sin\phi_o)\cos\theta\sin\psi - (W_t \cos\theta_o \cos\phi_o)\sin\theta$$

$$\begin{aligned} y_3 = {} & (-W_t \sin\theta_o)(\cos\psi \sin\theta \sin\phi - \sin\psi \cos\phi) \\ & + (W_t \cos\theta_o \sin\phi_o)(\cos\psi \cos\phi + \sin\psi \sin\phi \sin\theta) \\ & + (W_t \cos\phi_o \cos\theta_o)(\cos\theta \sin\phi) \end{aligned}$$

$$\begin{aligned} z_3 = {} & (-W_t \sin\theta_o)(\cos\psi \sin\theta \cos\phi + \sin\psi \sin\phi) \\ & + (W_t \cos\theta_o \sin\phi_o)(\sin\psi \sin\theta \cos\phi - \cos\psi \sin\phi) \\ & + (W_t \cos\theta_o \cos\phi_o)(\cos\theta \cos\phi) \end{aligned}$$

The left sides of the three force equations are composed of aerodynamic, thrust and gravity forces. Let F_x , F_y , and F_z be the first two. The six equations of motion are now:

$$\Sigma F_x = m(\dot{U} + QW - RV) - x_3$$

$$\Sigma F_y = m(\dot{V} + RU - PW) - y_3$$

$$\Sigma F_z = m(\dot{W} + PV - QU) - z_3$$

$$\Sigma L = \dot{P} I_x - \dot{R} J_{xz} + QR(I_z - I_y) - PQ J_{xz}$$

$$\Sigma M = \dot{Q} I_y + PR(I_x - I_z) - R^2 J_{xz} + P^2 J_{xz}$$

$$\Sigma N = \dot{R} I_z - \dot{P} J_{xz} + PQ(I_y - I_x) + QR J_{xz}$$

The Relation Between P, Q, and R; and $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$

The rotation of the Eulerian axes has been accounted for, and the angular velocities P, Q, and R have been used in the preceding equations. These are the angular velocities about the axes x_3, y_3, and z_3 respectively. The angular velocities $\dot{\phi}$, $\dot{\theta}$, and $\dot{\psi}$, however, are about the x_2, y_1, and z axes respectively which are not orthogonal. Therefore expressing P, Q, and R in terms of the latter angular velocities is complex. By resolution in the same manner as in the previous case of the forces, they become:

$$P = \dot{\phi} - \dot{\psi} \sin\theta$$

$$Q = \dot{\theta} \cos\phi + \dot{\psi} \sin\phi \cos\theta$$

$$R = \dot{\psi} \cos\phi \cos\theta - \dot{\theta} \sin\phi$$

The rates of change of the Eulerian angles can also be expressed as functions of the instantaneous angular velocities by solving the above equations to give:

$$\dot{\phi} = P + Q \tan\theta \sin\phi + R \tan\theta \cos\phi$$

$$\dot{\theta} = Q \cos\phi - R \sin\phi$$

$$\dot{\psi} = R\left(\frac{\cos\phi}{\cos\theta}\right) + Q\left(\frac{\sin\phi}{\sin\theta}\right)$$

Linearization of the Equations of Motion

The linear and angular velocities can be presented in terms of their initial or steady state values and disturbed increments.

$$U = U_0 + u \qquad P = P_0 + p$$

$$V = V_0 + v \qquad Q = Q_0 + q$$

$$W = W_0 + w \qquad R = R + r$$

Substituting these quantities into the equations of motion:

$$\begin{aligned}\sum F_x = m\Big[&(\dot{u} + Q_0 W_0 + W_0 q + Q_0 w + w q - V_0 R_0 - V_0 r - R_0 v - r v) \\ &+ (g \sin\theta_0)\cos\theta\cos\psi - (g\cos\theta_0 \sin\phi_0)\cos\theta\sin\psi \\ &+ (g\cos\theta_0 \cos\phi_0)\sin\theta\Big]\end{aligned}$$

$$\begin{aligned}\sum F_y = m\Big[&(\dot{v} + R_0 U_0 + U_0 r + R_0 u + r u - P_0 W_0 - W_0 p - P_0 w - p w) \\ &-(g\cos\theta_0 \sin\phi_0)(\cos\psi\cos\phi + \sin\psi\sin\theta\sin\phi) \\ &+(g\sin\theta_0)(\cos\psi\sin\theta\sin\phi - \sin\psi\cos\phi) \\ &-(g\cos\theta_0\cos\phi_0)(\cos\theta\sin\phi)\Big]\end{aligned}$$

$$\begin{aligned}\sum F_z = m\Big[&(\dot{w} + P_0 V_0 + V_0 p + P_0 v + p v - Q_0 U_0 - Q_0 u - U_0 q - q u) \\ &+(g\sin\theta_0)(\cos\psi\sin\theta\cos\phi + \sin\psi\sin\phi) \\ &-(g\cos\theta_0\sin\phi_0)(\sin\psi\sin\theta\cos\phi - \cos\psi\sin\phi) \\ &-(g\cos\theta_0\cos\phi_0)(\cos\theta\cos\phi)\Big]\end{aligned}$$

$$\sum L = \dot{p} I_x - \dot{r} J_{xz} + (Q_0 R_0 + R_0 q + Q_0 r + q r)(I_z - I_y) - (P_0 Q_0 + Q_0 p + P_0 q + p q) J_{xz}$$

$$\sum M = \dot{q} I_y + (P_0 R_0 + R_0 p + P_0 r + p r)(I_x - I_z) - (R_0^2 + 2 R_0 r + r^2) J_{xz} + (P_0^2 + 2 P_0 p + p^2) J_{xz}$$

$$\sum N = \dot{r} I_z - \dot{p} J_{xz} + (P_0 Q_0 + Q_0 p + P_0 q + p q)(I_y - I_x) - (Q_0 R_0 + R_0 q + Q_0 r + q r) J_{xz}$$

Disturbances from steady flight are assumed small enough that products and higher orders of velocity increments are negligible in comparison with the increments themselves. Also disturbance angles are sufficiently small that *sin* is approximated by the angle in radians and *cos* is approximated by unity. Further, products of these angles may be neglected. Finally any changes in air density may be assumed to be zero.

The equations of motion can now be reduced to:

$$\Sigma F_x = m\Big[(\dot{u} + Q_o W_o + W_o q + Q_o w - R_o V_o - R_o v - V_o r)$$
$$+ (g \sin\theta_o) - (g \cos\theta_o \sin\varphi_o)\psi + (g \cos\theta_o \cos\varphi_o)\theta\Big]$$

$$\Sigma F_y = m\Big[(\dot{v} + R_o U_o + U_o r + R_o u - P_o W_o - W_o p - P_o w)$$
$$- (g \sin\theta_o)\psi - g\cos\theta_o \sin\varphi_o - (g \cos\theta_o \cos\varphi_o)\varphi\Big]$$

$$\Sigma F_z = m\Big[(\dot{w} + P_o V_o + P_o v + V_o p - Q_o U_o - Q_o u - U_o q)$$
$$+ (g \sin\theta_o)\theta + (g \cos\theta_o \sin\varphi_o)\varphi - (g \cos\theta_o \cos\varphi_o)\Big]$$

$$\Sigma L = \dot{p} I_x - \dot{r} J_{xz} + (Q_o R_o + R_o q + Q_o r)(I_z - I_y) - (P_o Q_o + Q_o p + P_o q) J_{xz}$$

$$\Sigma M = \dot{q} I_y + (P_o R_o + R_o p + P_o r)(I_x - I_z) - (R_o^2 + 2R_o r) J_{xz} + (P_o^2 + 2P_o p) J_{xz}$$

$$\Sigma N = \dot{r} I_z - \dot{p} J_{xz} + (P_o Q_o + Q_o p + P_o q)(I_y - I_x) - (Q_o R_o + R_o q + Q_o r) J_{xz}$$

The assumption of small disturbances not only reduces the equations but also linearizes them so that their solution will be simpler. Although this assumption seems to limit the motions to infinitesimal disturbances from steady flight, experience has shown that finite deflections can be treated in this manner without serious error.

The equations of related angular changes have also been simplified:

$$P = \dot{\varphi} - \dot{\psi}\theta$$
$$Q = \dot{\theta} + \dot{\psi}\varphi$$
$$R = \dot{\psi} - \dot{\theta}\varphi$$

and neglecting products of small disturbances:

$$P = \dot{\phi} \quad , \qquad Q = \dot{\theta} \quad , \qquad R = \dot{\psi}$$

Thus the instantaneous angular velocities may be set equal to the rate of change of the Eulerian angles.

The equations of motion now allow the examination of the motion of a body disturbed from some complicated steady flight condition such as combined steady state rolling, pitching, yawing motion with constant sideslip and forward velocity.

In order to reduce the equations of motion still further to enable easier visualization, a steady flight condition is assumed with wings level and all components of velocity zero except U_o and W_o .

$$V_o = P_o = Q_o = R_o = \Phi_o = \psi_o = 0$$

The equations of motion now become:

$$\Sigma F_x = m\left[\dot{u} + W_o q + g \sin\theta_o + g\theta\cos\theta_o\right]$$

$$\Sigma F_y = m\left[\dot{v} + U_o r - W_o p - g\psi\sin\theta_o - g\phi\cos\theta_o\right]$$

$$\Sigma F_z = m\left[\dot{w} - U_o q + g\theta\sin\theta_o - g\cos\theta_o\right]$$

$$\Sigma L = \dot{p}\, I_x - \dot{r}\, J_{xz}$$

$$\Sigma M = \dot{q}\, I_y$$

$$\Sigma N = \dot{r}\, I_z - \dot{p}\, J_{xz}$$

So far the only specification as to the direction of the Eulerian axes has been that the y axis is a principal inertial axis of the aircraft. If the x axis is made to coincide with the

longitudinal axis of the aircraft, it is also a principal axis. This system is referred to as the "body" system of axes. The advantage of this system is that the value of J_{xz} is zero. However, if the x axis is made to coincide with the flight direction during steady flight, the resulting system is called the stability or "wind axes" system in which W_o terms vanish. The actual choice of axes will be made later.

Expansion of the Aerodynamic Forces and Moments

Each of the forces X, Y, and Z and the moments L, M, and N can be expressed as a function of the variables upon which they depend by expanding the forces in a Taylor series. These series have the form:

$$F = (F)_0 + \left(\frac{\partial F}{\partial \alpha}\right)_0 \alpha + \left(\frac{\partial F}{\partial \beta}\right)_0 \beta + \left(\frac{\partial F}{\partial \gamma}\right)_0 \gamma + \cdots$$

where α, β, and γ are variables; the subscript zero indicates that the quantities are evaluated at the steady flight conditions; and all second-order terms have been neglected in accordance with assumption V.

Because of the symmetry of the x, z plane, the rate of change of the X and Z forces and the moment M, with respect to the disturbance velocities p, r, and v, is equal to zero. Only derivatives with respect to u, w, q and their time derivatives are retained according to the level flight assumption. If, instead of this assumption, a different initial condition, such as steady sideslip, had been assumed, the derivatives would have to be reevaluated. Of course, $\left(\frac{\partial X}{\partial v}\right)_0$ would not have been zero.

The external forces are now expressed:

$$\sum F_x = X = X_0 + \frac{\partial X}{\partial u} u + \frac{\partial X}{\partial \dot{u}} \dot{u} + \frac{\partial X}{\partial q} q + \frac{\partial X}{\partial \dot{q}} \dot{q} + \frac{\partial X}{\partial w} w + \frac{\partial X}{\partial \dot{w}} \dot{w} + \frac{\partial X}{\partial \delta_e} \delta_e + \frac{\partial X}{\partial \dot{\delta}_e} \dot{\delta}_e + \frac{\partial X}{\partial \ddot{\delta}_e} \ddot{\delta}_e$$

$$\sum F_y = Y = y_0 + \frac{\partial Y}{\partial v} v + \frac{\partial Y}{\partial \dot{v}} \dot{v} + \frac{\partial Y}{\partial r} r + \frac{\partial Y}{\partial \dot{r}} \dot{r} + \frac{\partial Y}{\partial p} p + \frac{\partial Y}{\partial \dot{p}} \dot{p}$$
$$+ \frac{\partial Y}{\partial \delta_a} \delta_a + \frac{\partial Y}{\partial \dot{\delta}_a} \dot{\delta}_a + \frac{\partial Y}{\partial \ddot{\delta}_a} \ddot{\delta}_a + \frac{\partial Y}{\partial \delta_r} \delta_r + \frac{\partial Y}{\partial \dot{\delta}_r} \dot{\delta}_r + \frac{\partial Y}{\partial \ddot{\delta}_r} \ddot{\delta}_r$$

$$\sum F_z = Z = z_0 + \frac{\partial Z}{\partial u} u + \frac{\partial Z}{\partial \dot{u}} \dot{u} + \frac{\partial Z}{\partial q} q + \frac{\partial Z}{\partial \dot{q}} \dot{q} + \frac{\partial Z}{\partial w} w + \frac{\partial Z}{\partial \dot{w}} \dot{w}$$
$$+ \frac{\partial Z}{\partial \delta_e} \delta_e + \frac{\partial Z}{\partial \dot{\delta}_e} \dot{\delta}_e + \frac{\partial Z}{\partial \ddot{\delta}_e} \ddot{\delta}_e$$

$$L = L_0 + \frac{\partial L}{\partial v} v + \frac{\partial L}{\partial \dot{v}} \dot{v} + \frac{\partial L}{\partial r} r + \frac{\partial L}{\partial \dot{r}} \dot{r} + \frac{\partial L}{\partial p} p + \frac{\partial L}{\partial \dot{p}} \dot{p} + \frac{\partial L}{\partial \delta_a} \delta_a + \frac{\partial L}{\partial \dot{\delta}_a} \dot{\delta}_a$$
$$+ \frac{\partial L}{\partial \ddot{\delta}_a} \ddot{\delta}_a + \frac{\partial L}{\partial \delta_r} \delta_r + \frac{\partial L}{\partial \dot{\delta}_r} \dot{\delta}_r + \frac{\partial L}{\partial \ddot{\delta}_r} \ddot{\delta}_r$$

$$M = M_0 + \frac{\partial M}{\partial u} u + \frac{\partial M}{\partial \dot{u}} \dot{u} + \frac{\partial M}{\partial q} q + \frac{\partial M}{\partial \dot{q}} \dot{q} + \frac{\partial M}{\partial w} w + \frac{\partial M}{\partial \dot{w}} \dot{w}$$
$$+ \frac{\partial M}{\partial \delta_e} \delta_e + \frac{\partial M}{\partial \dot{\delta}_e} \dot{\delta}_e + \frac{\partial M}{\partial \ddot{\delta}_e} \ddot{\delta}_e$$

$$N = N_0 + \frac{\partial N}{\partial v} v + \frac{\partial N}{\partial \dot{v}} \dot{v} + \frac{\partial N}{\partial r} r + \frac{\partial N}{\partial \dot{r}} \dot{r} + \frac{\partial N}{\partial p} p + \frac{\partial N}{\partial \dot{p}} \dot{p}$$
$$+ \frac{\partial N}{\partial \delta_a} \delta_a + \frac{\partial N}{\partial \dot{\delta}_a} \dot{\delta}_a + \frac{\partial N}{\partial \ddot{\delta}_a} \ddot{\delta}_a + \frac{\partial N}{\partial \delta_r} \delta_r + \frac{\partial N}{\partial \dot{\delta}_r} \dot{\delta}_r + \frac{\partial N}{\partial \ddot{\delta}_r} \ddot{\delta}_r$$

In accordance with the level flight assumption, the following initial or steady flight quantities will be eliminated when the aerodynamic forces and moments are substituted into the equations of motion:

$$x_0 - mg \sin \theta_0 = 0$$
$$y_0 = 0$$
$$z_0 + mg \cos \theta_0 = 0$$
$$L_0 = 0$$
$$M_0 = 0$$
$$N_0 = 0$$

Further, the stability axes will be used for the Eulerian axes system so that terms containing W_0 will be eliminated since there will be no velocity initially perpendicular to the steady flight path.

Substituting the aerodynamic forces and moments into the linearized equations of motion and dividing by the coefficients of $\dot{u}$, $\dot{v}$, $\dot{w}$, $\dot{p}$, $\dot{q}$, and $\dot{r}$, the equations become:

Using the notation:

$$\frac{1}{m}\frac{\partial X}{\partial u} = X_u \qquad \frac{1}{m}\frac{\partial Y}{\partial v} = Y_v$$

$$\frac{1}{I_y}\frac{\partial M}{\partial \dot{u}} = M_{\dot{u}} \qquad \frac{1}{I_x}\frac{\partial L}{\partial v} = L_v$$

for stability derivatives.

$$\dot{u} + g\theta\cos\theta_0 = X_u u + X_{\dot{u}}\dot{u} + X_q q + X_{\dot{q}}\dot{q} + X_w w + X_{\dot{w}}\dot{w} + X_{\delta_e}\delta_e + X_{\dot{\delta}_e}\dot{\delta}_e + X_{\ddot{\delta}_e}\ddot{\delta}_e$$

$$\dot{v} + U_0 r - g\psi\sin\theta_0 - g\phi\cos\theta_0 = Y_v v + Y_{\dot{v}}\dot{v} + Y_r r + Y_{\dot{r}}\dot{r} + Y_p p + Y_{\dot{p}}\dot{p} + Y_{\delta_a}\delta_a + Y_{\dot{\delta}_a}\dot{\delta}_a + Y_{\ddot{\delta}_a}\ddot{\delta}_a + Y_{\delta_r}\delta_r + Y_{\dot{\delta}_r}\dot{\delta}_r + Y_{\ddot{\delta}_r}\ddot{\delta}_r$$

$$\dot{w} - U_0 q + g\theta\sin\theta_0 = Z_u u + Z_{\dot{u}}\dot{u} + Z_q q + Z_{\dot{q}}\dot{q} + Z_w w + Z_{\dot{w}}\dot{w} + Z_{\delta_e}\delta_e + Z_{\dot{\delta}_e}\dot{\delta}_e + Z_{\ddot{\delta}_e}\ddot{\delta}_e$$

$$\dot{p} - \frac{J_{xz}}{I_x}\dot{r} = L_v v + L_{\dot{v}}\dot{v} + L_r r + L_{\dot{r}}\dot{r} + L_p p + L_{\dot{p}}\dot{p} + L_{\delta_a}\delta_a + L_{\dot{\delta}_a}\dot{\delta}_a + L_{\ddot{\delta}_a}\ddot{\delta}_a + L_{\delta_r}\delta_r + L_{\dot{\delta}_r}\dot{\delta}_r + L_{\ddot{\delta}_r}\ddot{\delta}_r$$

$$\dot{q} = M_u u + M_{\dot{u}}\dot{u} + M_q q + M_{\dot{q}}\dot{q} + M_w w + M_{\dot{w}}\dot{w} + M_{\delta_e}\delta_e + M_{\dot{\delta}_e}\dot{\delta}_e + M_{\ddot{\delta}_e}\ddot{\delta}_e$$

$$\dot{r} - \frac{J_{xz}}{I_z}\dot{p} = N_v v + N_{\dot{v}}\dot{v} + N_r r + N_{\dot{r}}\dot{r} + N_p p + N_{\dot{p}}\dot{p} + N_{\delta_a}\delta_a + N_{\dot{\delta}_a}\dot{\delta}_a + N_{\ddot{\delta}_a}\ddot{\delta}_a + N_{\delta_r}\delta_r + N_{\dot{\delta}_r}\dot{\delta}_r + N_{\ddot{\delta}_r}\ddot{\delta}_r$$

Here another simplifying assumption is made that the forces and moments acting on the aircraft are functions of the velocities and not the changes in the velocities relative to the air mass. This is the "instantaneous" view of the forces acting on an accelerating aircraft as if it were in steady flight with velocities at a given instant. This is also referred to as "quasi steady flow."

Assuming quasi steady flow, all derivatives with respect to the rates of change of velocity, both linear and angular (except those involving $\dot{w}$ and $\dot{v}$) and all those with respect to control deflection accelerations will be eliminated from the equations of motion. The vertical acceleration, $\dot{w}$, and the lateral acceleration, $\dot{v}$, are retained because of the effects of downwash and sidewash, respectively, from the wing and fuselage on the tail, which may be considered from the quasi steady flow viewpoint.

Consider an aircraft at two instances of time, t_1 , and t_2 , such that $w_1 \neq w_2$. Assuming $w_2 > w_1$, the wing will experience an increase in α at t_2 such that $\Delta\alpha = \tan^{-1}\frac{\Delta w}{U_0}$ (U_0 = flight speed)

For small Δt , $w_2 = w_1 + \frac{dw}{dt}\Delta t = w_1 + \dot{w}\Delta t$.

The aircraft moves forward a distance $U_0 \Delta t$ in the time interval, and the tail will reach the position vacated by the wing in the time $\frac{l_t}{U_0}$, where l_t tail length.

Thus:

$$w_2 = w_1 + \dot{w}\frac{l_t}{U_0}$$

Now the change in angle of attack of the tail is proportional to the change in downwash which is assumed to change instantaneously in accordance with the quasi steady theory.

Obviously the forces and moments on the aircraft are affected by the angle of attack of the tail which in turn is affected by $\dot{w}$ on the basis of the quasi steady flow method. It is for this reason that derivatives with respect to $\dot{w}$ are retained in the equations of motion.

The effect of sidewash on the vertical tail can be treated similarly and therefore derivatives with respect to $\dot{v}$ are also retained.

The reduced equations of motion become:

Longitudinal:

$$\dot{u} + g\theta\cos\theta_0 = X_u u + X_q q + X_w w + X_{\dot{w}} \dot{w} + X_{\delta e}\delta e$$

$$\dot{w} - U_0 q + g\theta\sin\theta_0 = Z_u u + Z_q q + Z_w w + Z_{\dot{w}} \dot{w} + Z_{\delta e}\delta e$$

$$\dot{q} = M_u u + M_q q + M_w w + M_{\dot{w}} \dot{w} + M_{\delta e}\delta e$$

Lateral:

$$\dot{v} + U_0 r - g\psi\sin\theta_0 - g\phi\cos\theta_0 = Y_v v + Y_{\dot{v}}\dot{v} + Y_r r + Y_p p + Y_{\delta a}\delta a + Y_{\delta r}\delta r$$

$$\dot{p} - \frac{J_{xz}}{I_x}\dot{r} = L_v v + L_{\dot{v}}\dot{v} + L_r r + L_p p + L_{\delta a}\delta a + L_{\delta r}\delta r$$

$$\dot{r} - \frac{J_{xz}}{I_z}\dot{p} = N_v v + N_{\dot{v}}\dot{v} + N_r r + N_p p + N_{\delta a}\delta a + N_{\delta r}\delta r$$

The expansion of the determinant of the longitudinal equations of motion in terms of an assumed exponential root, λ , leads to a

fourth order equation of the form:

$$A\lambda^4 + B\lambda^3 + C\lambda^2 + D\lambda + E = 0$$

which is known as the stability quartic. A similar quartic results from the expansion of the determinant of the lateral equations of motion. It is these equations which the stability model's analog system must solve. In the following section the computation methods used in the trainer are examined to see if they can accommodate the stability quartic.

Solution by Analog Computer

In order to simulate the motion and response of an aircraft, it will be necessary to employ an analog computer which will accept the pilot's control signals as primary variable inputs and the stability derivatives as secondary "fixed" inputs. From the primary and secondary inputs, simulated position, rates and accelerations of motion are computed.

Typical schematics of the six sections of an aircraft motion analog computer are shown in Figures 7 through 12.

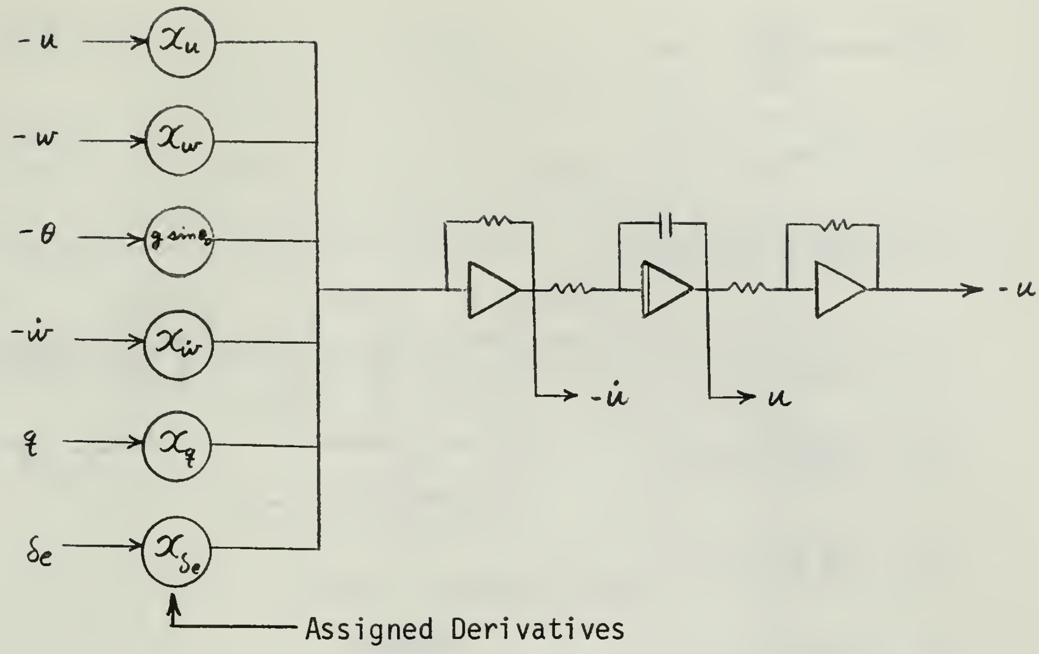


Figure 7

Analog Longitudinal Motion

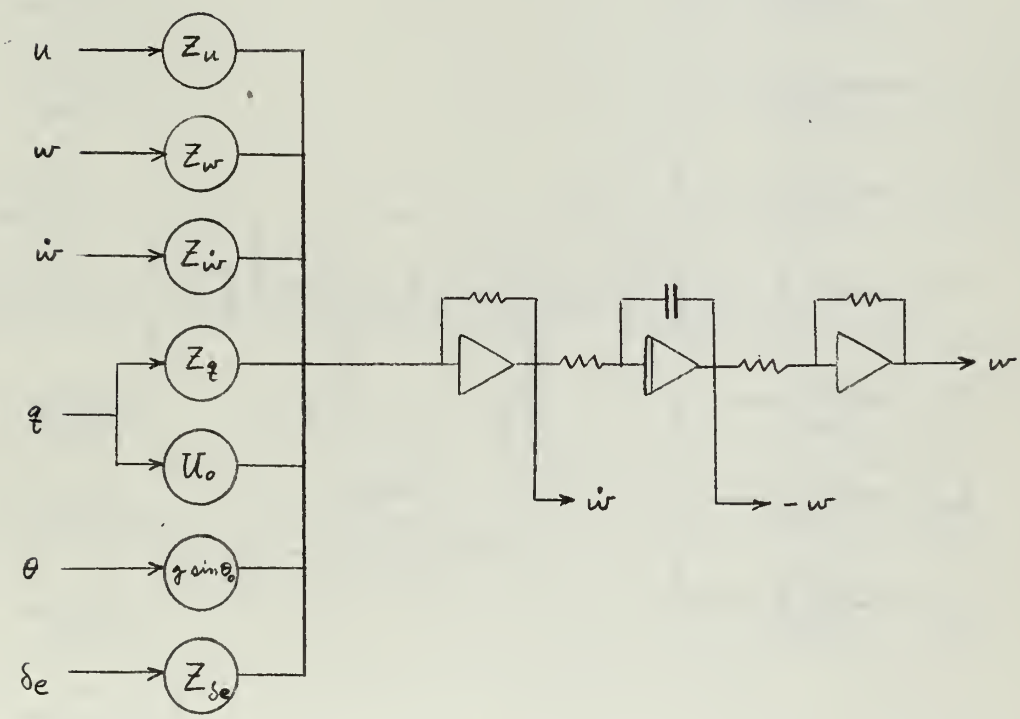


Figure 8

Analog Vertical Motion

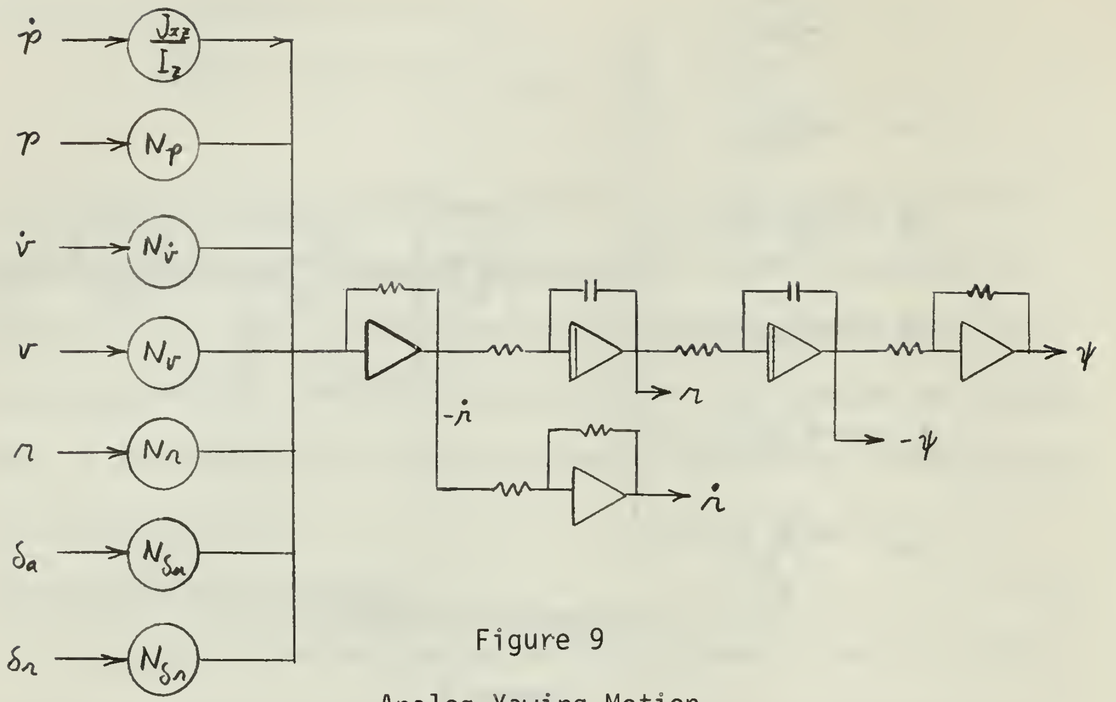


Figure 9

Analog Yawing Motion

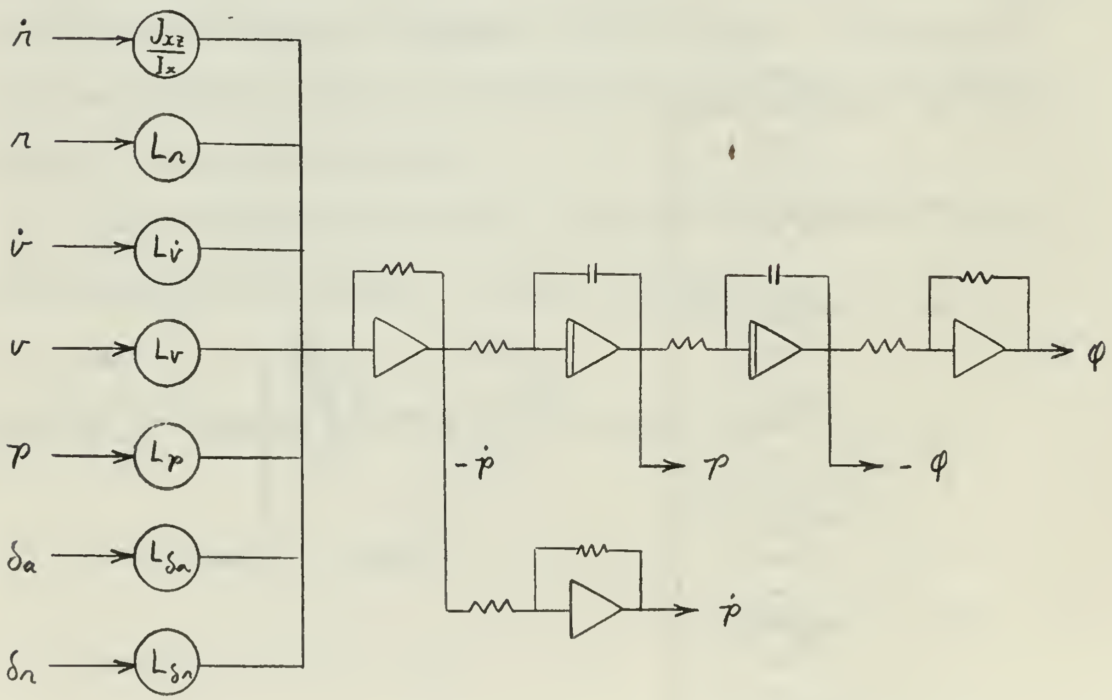


Figure 10

Analog Rolling Motion

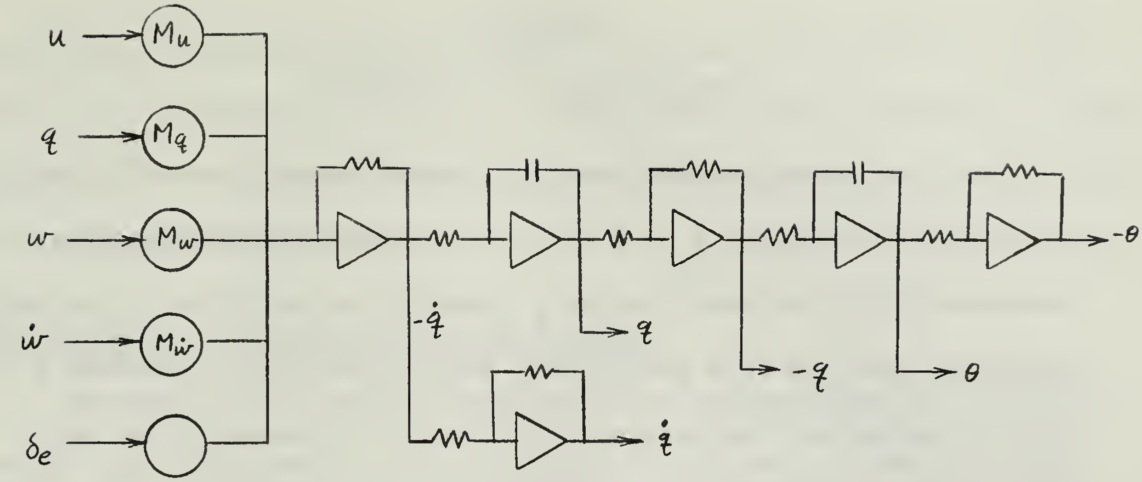


Figure 11

Analog Pitching Motion

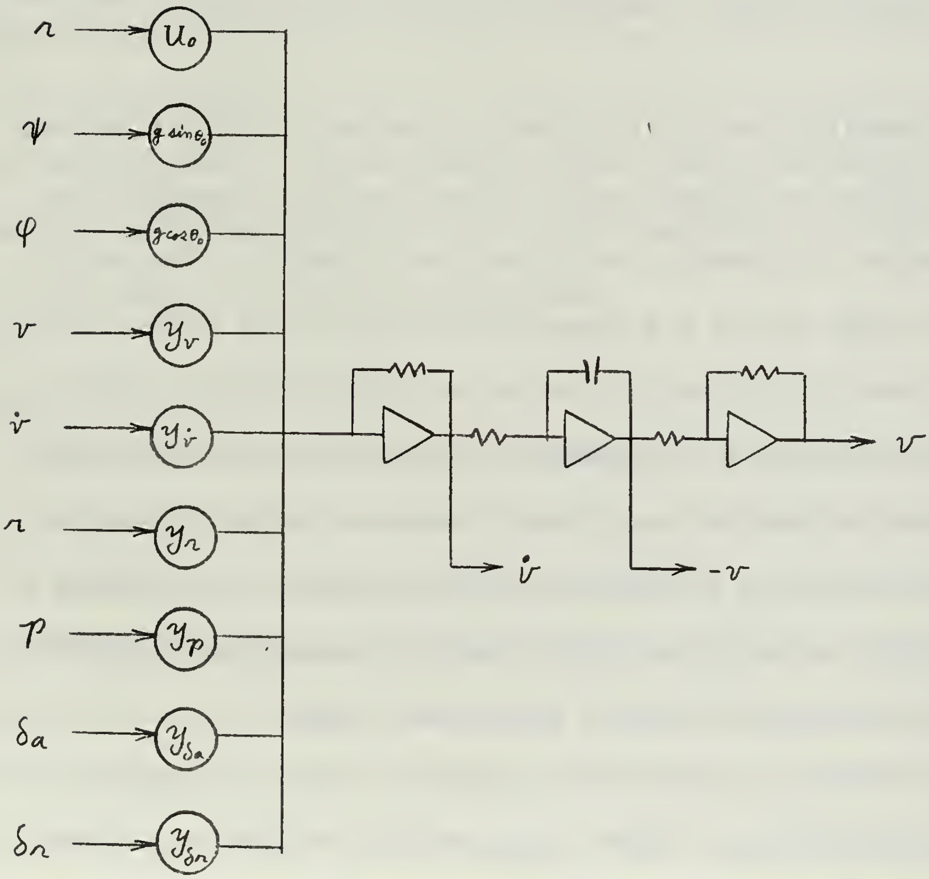


Figure 12

Analog Lateral Motion

CHAPTER III

DESCRIPTION OF THE TRAINER[3]

The "Jet-Propelled Aircraft Instrument Flying Trainer," Type C-11, is designed so that most of its flight, engine and communications characteristics are electronically computed and activated. The basic structure consists of a metal frame which supports the fuselage with cockpit mockup and the instructor's console. Components and sub-assemblies are mounted on sliding racks which can be pulled out from the framework for maintenance and adjustment. The framework itself is sectionalized into four main parts which are interconnected with cabling of sufficient length to allow the trainer to remain operationally connected while separated for maintenance work on the interior.

The cockpit of the C-11 trainer is basically a mockup of the F-80 jet aircraft with appropriate flight, engine, communications and navigation instruments, indicators and controls. The control stick and rudder pedals are pneumatically restricted in their movements to simulate the control loading of a jet aircraft. These controls are connected to pneumatic cylinders and pistons in which the pressure is governed by a dynamic pressure factor related to airspeed, air density (altitude), and Mach number. The controls including the throttle are also connected to potentiometers which provide the control commands as electrical signals.

The instructor's console has duplicate flight and engine instruments and indicator lights which monitor the function of the computers. There is also a ground recorder which plots the simulated track of the aircraft over the ground calibrated to a

sectional aeronautical chart. Another recorder plots the glide path of the aircraft during simulated landing approaches. The console also has controls for governing the simulated communications and navigation environment of the trainer.

The Methods of Computation in the Trainer[4]

Aerodynamic and engine characteristics are computed by electronic relay and electro-mechanical computers. These are mounted on sliding racks underneath the cockpit and behind the console.

The mathematical processes of multiplication, addition, division and integration are accomplished by a network of potentiometers, resolvers, servos and amplifiers to produce voltages or shaft positions equivalent to the desired factors. Signal voltages are used to control servo amplifiers which, in turn, control the position or speed of servo motors. The servo motors then control the selsyns, resolvers and potentiometers used in the computation of other factors or directly as instrument indications.

Longitudinal Acceleration

Longitudinal acceleration is produced by all the aerodynamic and ground forces acting on the aircraft along its longitudinal axis in the direction of the flight path and opposite to the relative wind. Deceleration is produced by drag coefficients such as profile drag, wing flaps, dive flaps, landing gear, angle of attack and forces developed while in a spin. While on the ground, additional deceleration of brake forces is added. These decelerative forces are further increased by the horizontal component of path elevation angle, and the factor proportional to the summation of all drag coefficients is modified by dynamic pressure. The factor proportional

to net drag is added to the factor proportional to net thrust; and the result, indicating accelerative or decelerative force, is applied to the true velocity servo where its integration with respect to time results in true velocity.

A voltage proportional to airborne drag is derived by the summation of the amplitudes of voltages proportional to angle of attack, spin, profile drag, wing flap position, speed brakes position and landing gear position. These voltages, identical in phase but varying in amplitude in proportion to their effect upon total drag, are added together in the longitudinal acceleration summing amplifier.

Lateral Acceleration

Lateral acceleration is the acceleration of the aircraft along the lateral axis due to all aerodynamic and ground forces except gravity. In the air, lateral acceleration is considered to be a function of dynamic pressure, engine net thrust and yaw angle. When the aircraft is on the ground, lateral acceleration is proportional to the difference in brake pedal deflection, rudder deflection and dynamic pressure.

In straight and level flight, the angle of yaw is zero, and therefore lateral acceleration is also zero. If, however, the aircraft is yawed, dynamic pressure and engine thrust produce lateral acceleration that is directly proportional to the magnitude of the yaw angle.

Computer voltage proportional to dynamic pressure, engine thrust, rudder deflection, differential brake deflections, and angle of yaw are combined. The resultant voltage, representing the tendency of the aircraft to skid, slip or turn is made available to the heading, rate of turn and ball-bank computers.

Vertical Aerodynamic Acceleration

Acceleration along the vertical axis with respect to the relative wind is produced by all the aerodynamic and ground forces in addition to the gravitational force. The vertical aerodynamic computer in the trainer calculates the lift factor of the aircraft from angle of attack, wing flap position, path elevation angle, and dynamic pressure. Relays and phase detectors are so arranged that the lift force component must equal the weight component when landing, or an enforced take-off will occur with resultant bumpy landing. The vertical acceleration component is routed to the pitching rate, ball-bank, heading, path elevation angle, angle of attack, and accelerometer computers and is used in their various computations.

Normal Acceleration

The accelerometer system uses a single pointer type indicator which registers instantaneous "g." A voltage from the vertical aerodynamic acceleration computer, which varies in magnitude, is applied to the accelerometer servo amplifier which controls a servo motor. Coupled to the motor through a gear train is the pointer on the acceleration indicator. The magnitude of the voltage from the vertical acceleration computer is proportional to "g." The accelerometer registers between zero and plus 9.5 "g." Negative "g" is not indicated.

Angle of Attack

The angle of attack is the angle between the longitudinal axis of the aircraft and the relative wind. Angle of attack is computed by adding together voltages representing vertical aerodynamic acceleration and the vertical component of bank angle, modifying this

sum by path elevation angle, and further modifying the whole by true velocity. The product of these factors is added to a voltage representing pitch rate and is applied to the angle of attack servo amplifier, which in turn drives the angle of attack servo motor. The angle of attack servo functions as a velocity servo. A damping amplifier is included in the computer to improve the simulated longitudinal stability.

Angle of Bank

Six factors are simultaneously active in computing angle of bank or roll. Aileron deflection and angle of yaw are the primary and most important factors entering into the function. Both of these are more effective at high air speed and are considered as varying directly in proportion to true velocity. The effect of unbalanced lift in a turn due to the difference in relative speed of the wings enters as a factor proportional to the rate of turn. This last factor is the effect of a change of flight path azimuth when path elevation angle is other than zero. The angle of bank computer functions as a velocity servo system when the aircraft altitude is greater than field elevation. The mechanical output is used to drive resolvers in the heading, bank, and Zero Reader vertical pointer computers and to operate sensing switches in the polar cap sensor computer.

Pitching Rate

The pitching rate is proportional to the summation of all factors which normally affect the pitch angle. Factors proportional to angle of attack, elevator deflection, wing flap deflection, speed brake extension, landing gear position, true velocity, and Mach

number are added together, and their sum is added to factors representing net thrust and the gravitational component of vertical acceleration. This sum is further modified by dynamic pressure and applied to the pitching rate servo amplifier. The pitching rate servo functions as a velocity servo driving two potentiometers, one in its own computer system and the other which serves as a factor in computing angle of attack.

Angle of Yaw

The yaw angle is the angle between the X,Z plane of the aircraft and the tangent to the flight path due to its rotation about the vertical axis. Yaw is affected by rudder and/or aileron deflection. Voltages proportional to these factors are combined and utilized as the initiating voltage for the yaw servo on the heading servo chassis. The yaw servo drives potentiometers affecting the bank angle and lateral acceleration computers. It also provides yaw function to the heading computer through a mechanical differential on the heading servo.

Flight Path Elevation Angle

The flight path elevation angle is between the tangent to the flight path and the horizontal plane. It is computed by adding the vertical components of bank and pitch angles and modifying their sum by true velocity. This output is utilized as an initiating voltage for the path elevation angle servo, functioning as a velocity servo when the aircraft is on the ground. The servo actuates switches, potentiometers, and resolvers in other computers requiring the path elevation angle in their computations.

Vertical Velocity

The vertical speed computer functions as a position servo. Its input is a voltage derived from the altitude computer which is proportional to the vertical component of true velocity. Its amplitude indicates the rate of vertical velocity, and its phase indicates the direction, either climb or dive.

Altitude

The altitude system computes simulated altitude by integrating the vertical speed component of true velocity with respect to time. A servo motor, whose speed and direction of rotation are proportional to the amplitude and phase of the vertical speed voltage, gives a shaft position representing the vertical speed component. The servo motor shaft drives various potentiometers and transmitters which provide inputs to other systems utilizing functions of altitude in their computations. A switching system on the console panel is incorporated to enable the instructor to control the altitude of field elevation.

Mach Number

Mach number is the ratio of the aircraft velocity along the flight path to the speed of sound at a given temperature and density. Temperature and density are functions of altitude. The Mach number computer combines altitude and velocity voltage factors, making a comparison to a reference voltage theoretically representing the speed of sound. A voltage equivalent to this ratio is derived and applied to the pitching rate computer when the Mach number exceeds 0.75.

Dynamic Pressure

The dynamic pressure is a function of air density and the square of the velocity. It is computed by modifying a voltage proportional to the ratio of air density at sea level to the air density at aircraft altitude by a voltage proportional to the square of the velocity. The dynamic pressure signal then drives a servo which positions potentiometers in other computers. The dynamic pressure servo also positions a valve in the control loading system regulating the forces on the control stick and rudder pedals.

It should be noted that air temperature or Mach number is not considered in this computation of dynamic pressure. This is a serious omission when flight velocity approaches or exceeds Mach 1.

A sufficient number of systems have been examined for an evaluation of the trainer in Chapter IV.

CHAPTER IV

EVALUATION OF THE TRAINER

Although the cockpit is a mockup of the F-80 aircraft, the parameters and stability model in the trainer, as is, do not truly simulate any particular aircraft. In fact, the trainer does not really simulate an aircraft; it only approximates one. The primary purpose of the trainer is training and proficiency in instrument navigation procedures and not the handling of a simulated aircraft. In the interest of economy of both cost and maintenance, the flight and engine computers are greatly simplified. Most functions are simply "on or off" depending on whether they are needed or not. Other functions are linear with any non-linear functions being represented by linear segments. The problem of solving the differential quartic stability equation, which is the simultaneous solution of the equations of motion, is avoided by this simplified "curve following" technique in the trainer.

Most of the hardware in the trainer consists of communications and navigation equipment which will not contribute to the variable stability model and can be discarded.

The cockpit mockup with its controls and instruments can be the basis for the simulator portion of the variable stability model. Only versatility modifications are necessary to make it easier for different types of pilot controls and indicator presentations to be evaluated in future projects.

It will be necessary to design and construct an analog computer which will be able to solve the equations of motion that have been developed here. Although none of the existing computers can be utilized, they will furnish a supply of component parts and subassemblies which might be used in constructing the analog circuits.

The power supplies presently in the trainer appear to be adequate to support any new equipment which will have to be constructed. They accept either single phase, 230 volt, 60 hertz or three phase, 208 volt, 60 hertz and produce a comprehensive assortment of AC and DC voltages.

The parameters which are fixed in the existing computer systems are buried in the circuits and often in several different places due to the piecemeal methods of computation. This precludes bringing them out to a panel and replacing them with only one variable potentiometer each. This means that a potentiometer panel to handle the variable parameter inputs can only be constructed in addition to the new analog system.

The pneumatic system which provides control force reactions to the pilot is controlled by a factor which is directly proportional to the computed dynamic pressure. In order to provide more accurate control force simulation, this factor should be related to the respective hinge moments that are developed and thereby subject to any parameter variation. This is not considered to be too important since it is anticipated that most of the future projects using the stability model as a simulator will be concerned with full power control systems.

The trainer also lacks two more systems which are essential in an effective simulator; a simulated visual terrain reference outside the cockpit and a cockpit motion system.[5]

The visual reference is absent because the trainer is concerned strictly with instrument flight. Normally, however, a pilot relies to a great extent on the visual cues he receives from the geography over which he is flying as to attitude and direction.

This can be simulated by a display on a projection screen which can be a television monitor. The picture displayed can be a signal from a television camera mounted with six degrees of freedom over a scale terrain model while the relative motion between the two is governed by the analog solution of the aircraft equations of motion. By another method, the picture displayed could be generated by a graphical spatial analysis computer which can provide the illusion of three dimensional motion over a grid surface. What this latter method lacks in realism it makes up for by having infinite geographical limits.

To be accurate, the cockpit of a simulator must also be provided with the same six degrees of freedom as the actual aircraft regarding accelerations; three angular and three translational. However, this is precluded by space and cost requirementsshort of the real aircraft. The economy of the trainer is such that there is no cockpit motion at all. This omission leaves dependence on visual cues only and removes the pilot's tactile cues of acceleration which lead the visual cues by a phase difference factor. The visual cues lag because they depend on displacement, the second integral of acceleration. The tactile cue is the actual acceleration.

There remains the spatial limitations imposed upon the excursions of the simulator cockpit. A dependence on visual cues only requires considerable displacements even when a scale model is used.

Because the tactile cues lead, the pilot uses them as a sort of early warning for the impending visual changes. Continued motion is not necessary because after the initial tactile cue the visual cues take over. Such has been established in tests of various simulation methods at the Ames Research Center, Moffett Field, California. It was further found that instead of a true acceleration

cue and exponential washout or decay of such a cue will provide a sufficient tactile cue. The actual duration of the acceleration can be reduced. The "lurch" felt by the pilot becomes a "nudge" as the exponential time constant is reduced, and the actual excursions required by the simulator cockpit can be kept within reasonable proportions.

When necessary, certain prolonged accelerations, such as the take-off roll, can be simulated by tilting the simulator cockpit slightly while keeping the relative visual attitude cues as if still level.

The trainer can be modified to incorporate a tactile cue system by mounting the simulator cockpit frame on a system of hydraulic jacks whose valves are actuated by signals from the analog motion computer.

A general schematic of the final form of the variable stability model and simulator is shown in Figure 13.

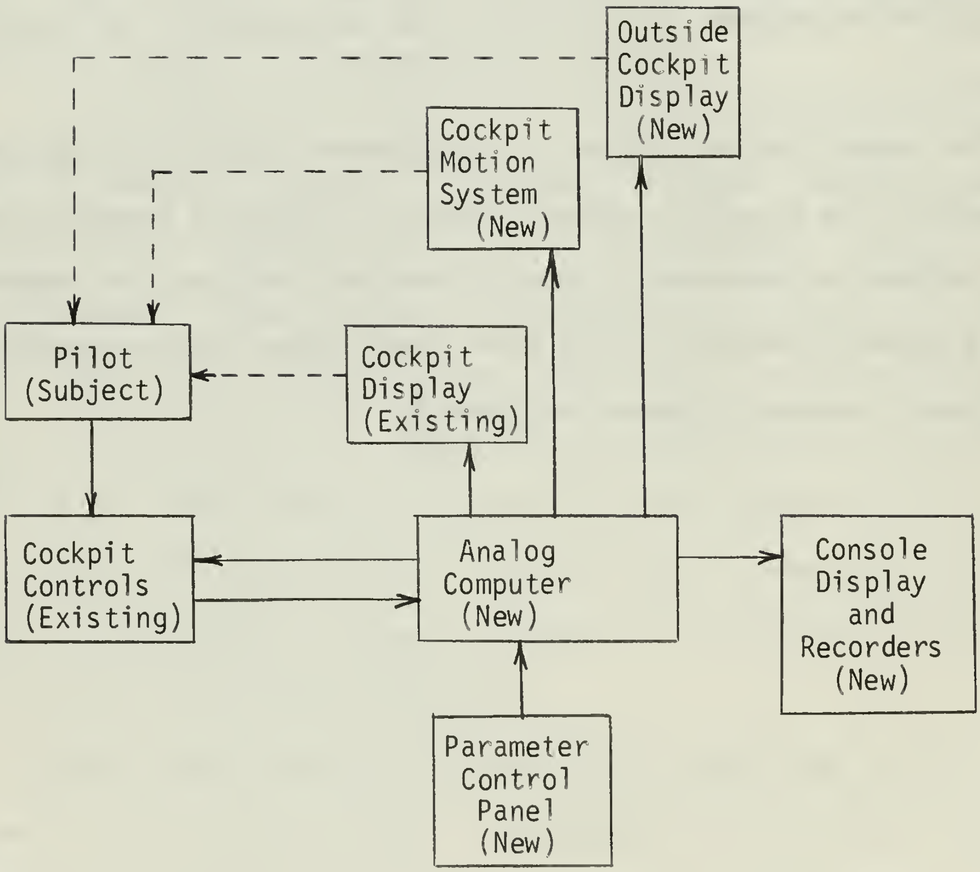


Figure 13

Aircraft Variable Stability Model and Simulator

CHAPTER V

CONCLUSIONS

The C-11 Instrument Flight Trainer cannot easily be converted into a variable aircraft stability model. However, the Trainer does have a suitable cockpit mockup with controls and indicators which can serve as the simulator portion of the stability model. The existing computers in the trainer will have to be replaced by an analog system which is capable of solving the stability quartic of the aircraft dynamic equations of motion. The detailed design of such an analog is recommended as a subject for another thesis as a sequel to this study.

The removal of the navigation, communications and computation equipment which is not needed for the stability model will make room for some of the new systems. The additional external features which will have to be constructed are: a potentiometer panel, to provide the inputs of the aircraft parameters which make up the stability derivatives necessary to the equations of motion; recording equipment, to plot the time histories of the resulting factors of motion; a cockpit motion system, to provide tactile cues to the pilot; and an outside cockpit display, to provide the pilot with a visual geographical reference.

The existing power supplies are vacuum tube types which could possibly be utilized by the new equipment unless solid state circuits are preferred. Solid state systems would be more reliable and require less cooling than vacuum tube systems.

The variable stability model and simulator will be a very useful tool for the investigation of stability and control problems. Transfer functions for human pilots as well as automatic controls can be evaluated.

REFERENCES

1. Scott, Dr. John E. Jr., Notes on Aircraft Equations of Motion. University of Virginia, School of Engineering and Applied Sciences, Charlottesville, Va. 1960 (unpublished)

2. Andrews, E.J., Automatic Throttle-Control for Deck-Recovery of Carrier-Based Airplanes. Naval Postgraduate School, Monterey, Calif. 1966 (unpublished)

3. USAF T.O.43D5-5-11, Instrument Flying Trainer, Jet-propelled Aircraft, Type C-11B, Mar 1964

4. C-11 Formulas and Curves, Link Aviation Devices, Inc., Binghamton, N.Y. 1950 (unpublished)

5. Andrews, E.J., The Importance of Motion Generation in the Operational Training Role of the Airplane Flight Simulator. Naval Postgraduate School, Monterey, Calif. 1967 (unpublished)

APPENDIX I

SYMBOLS USED IN C-11 INSTRUMENT FLIGHT TRAINER SCHEMATICS

B	Dynamic pressure substitute.
M	Mach number.
V_p	Airplane velocity along the flight path.
α	Angle of attack.
δ_{fw}	Fraction of wing flap deflection.
δ_{LG}	Fraction of full wheels down position.
δ_{fd}	Fraction of dive flap deflection.
δ_{Br}	Fraction of full right toe brake.
δ_{Bl}	Fraction of full left toe brake.
T_N	Net thrust.
F_x	Total longitudinal aerodynamic and ground force.
a_x	Total longitudinal acceleration.
γ	Angle of yaw.
δ_r	Rudder deflection.
F_y	Total lateral aerodynamic and ground force.
a_y	Total lateral acceleration.
F_{zG}	Vertical ground force on aircraft.
F_z	Total vertical aerodynamic and ground force.
a_z	Total vertical acceleration.
δ_e	Elevator deflection.
a_{zG}	Acceleration due to ground forces.
q_1	Pitching rate.
M_{ya}	Aerodynamic pitching moment.
I_y	Aircraft moment of inertia about the lateral axix.
g	Acceleration due to gravity.

β Flight path elevation angle.

ϕ Angle of bank.

h Altitude.

Constants used in C-11 Instrument Flight Trainer:

K_x = -7.8 for between -10 and 13.8 degrees.

= -26 for between 13.8 and 20 degrees.

K_y = -8.8 for between -10 and 13.8 degrees.

= 242. for between 13.8 and 20 degrees

K_1 = 18.2

K_2 - 45.

K_3 = 2.1

K_4 = 32.

K_5 = 750.

K_6 = Adjustable in the range 1.0 to 0.25

K_7 = 718.

K_8 = 958.

K_9 = 323

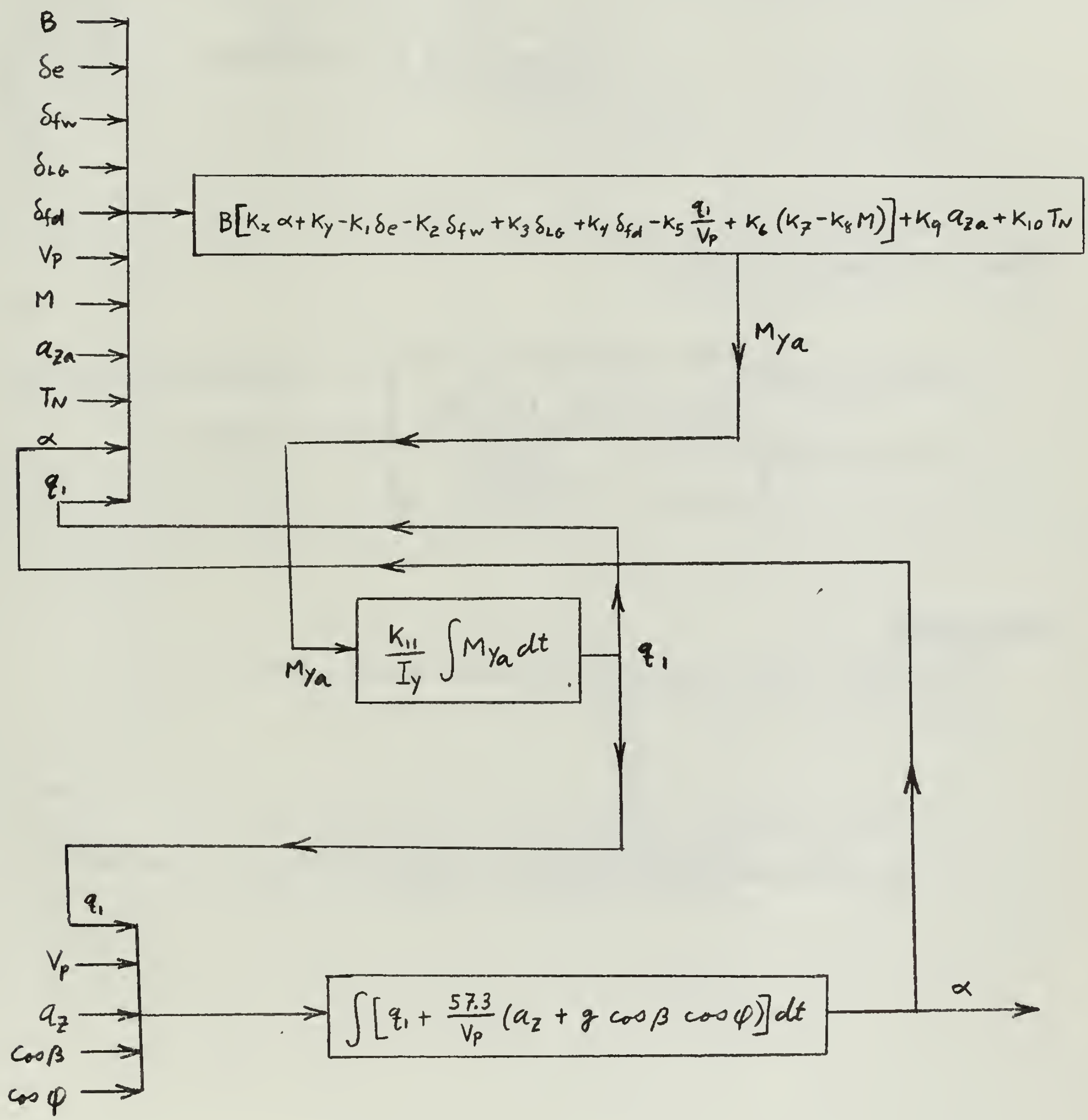
K_{10} = 0.4

K_{11} = 57.3

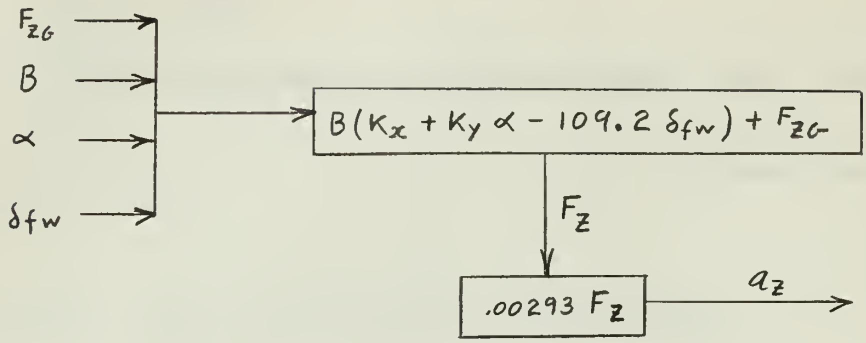
APPENDIX II

C-11 INSTRUMENT FLIGHT TRAINER COMPUTATION SCHEMATICS

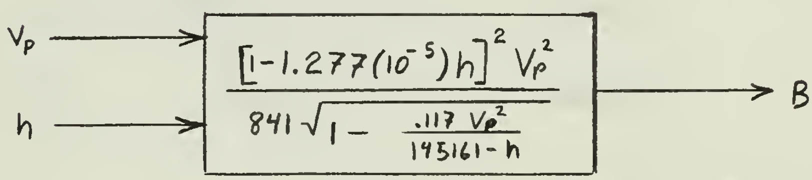
ANGLE OF ATTACK COMPUTER



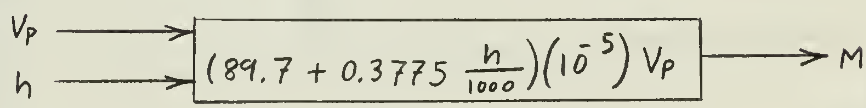
VERTICAL ACCELERATION



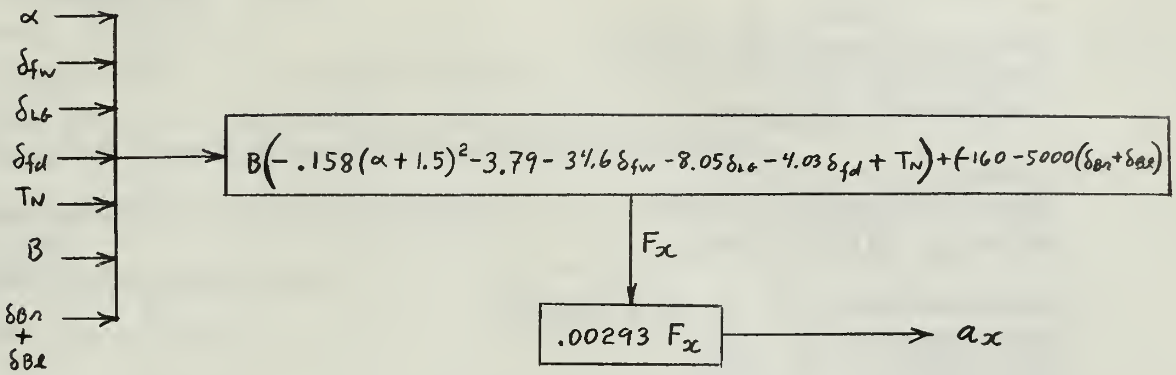
DYNAMIC PRESSURE SUBSTITUTE



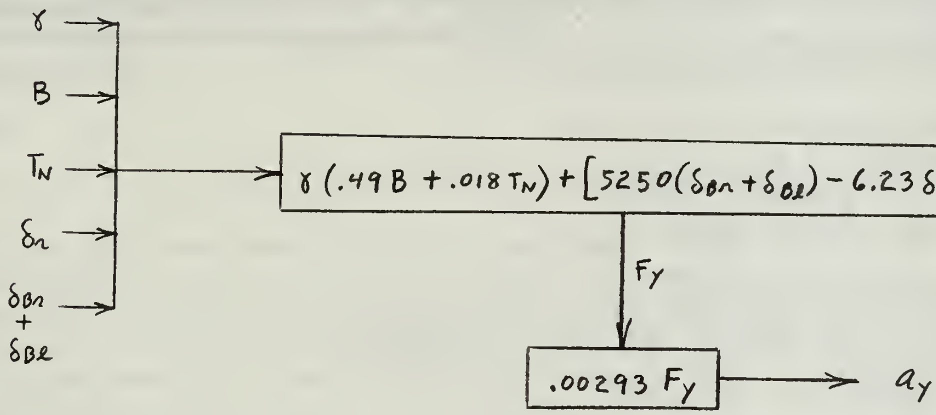
MACH NUMBER



LONGITUDINAL ACCELERATION



LATERAL ACCELERATION



INITIAL DISTRIBUTION LIST

		No Copies
1.	Defense Documentation Center Cameron Station Alexandria, Va. 22314	20
2.	Library Naval Postgraduate School Monterey, Calif. 93940	2
3.	Commander Naval Air Systems Command Department of the Navy Washington, D. C. 20360	1
4.	Professor E.J. Andrews (Thesis Advisor) Department of Aeronautics Naval Postgraduate School Monterey, Calif. 93940	2
5.	Cdr D.M. Layton, USN (Thesis Advisor) Department of Aeronautics Naval Postgraduate School Monterey, Calif. 93940	2
6.	Chairman, Department of Aeronautics Naval Postgraduate School Monterey, Calif. 93940	1
7.	Dr. A.E. Fuhs Department of Aeronautics Naval Postgraduate School Monterey, Calif. 93940	1
8.	University of Virginia School of Engineering and Applied Sciences Thorton Hall Charlottesville, Va. 22903	1
9.	Lt J. A. Johnson, USNR Rt 2, Box 3267 Ft. Pierce, Fla. 33450	3
10.	Lt C. J. Sweeney, USN SMC Box 1449 Naval Postgraduate School Monterey, Calif. 93940	1

Unclassified

Security Classification

DOCUMENT CONTROL DATA - R&D

(Security classification of title, body of abstract and indexing annotation must be entered when the overall report is classified)

1. ORIGINATING ACTIVITY *(Corporate author)*

Naval Postgraduate School,
Monterey, California

2a. REPORT SECURITY CLASSIFICATION

Unclassified

2b. GROUP

-

3. REPORT TITLE

STUDY OF A C-11 INSTRUMENT FLIGHT TRAINER AS A VARIABLE STABILITY MODEL AND SIMULATOR FOR STUDIES IN FLIGHT DYNAMICS

4. DESCRIPTIVE NOTES *(Type of report and inclusive dates)*

Thesis

5. AUTHOR(S) *(Last name, first name, initial)*

JOHNSON, John Arnold

6. REPORT DATE	7a. TOTAL NO. OF PAGES	7b. NO. OF REFS
December 1967	58	5

8a. CONTRACT OR GRANT NO.

b. PROJECT NO. NA

c.

d.

9a. ORIGINATOR'S REPORT NUMBER(S)

NA

This document has been approved for public release and sal..., its distribution is unlimited.

9b. OTHER REPORT NO(S) *(Any other numbers that may be assigned this report)*

None

10. AVAILABILITY/LIMITATION NOTICES

~~This document is subject to special export controls and each transmittal to foreign government or foreign nationals may be made only with prior approval of the U.S. Naval Postgraduate School~~.

Men 5/27/71

11. SUPPLEMENTARY NOTES

None

12. SPONSORING MILITARY ACTIVITY

Naval Postgraduate School, Monterey

13. ABSTRACT

A study has been made of a C-11 Instrument Flight Trainer to investigate its capabilities as a variable stability model and simulator. The cockpit, with its controls and instruments, and the power supplies were found to be suitable for this purpose. All other systems were not. The existing computers have been simplified to handle only linearized approximations of the aircraft stability equations of motion. An analog computer will have to be designed to solve the equations of motion in higher order differential form. In addition, systems will be required to provide simulated cockpit motion and a visual terrain reference.

DD FORM 1 JAN 64 1473

Unclassified

Security Classification

14 KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Aircraft Simulator						
Aircraft Stability Model						
C-11 Instrument Flight Trainer						
Stability Quartic, Analog Solution of						
Simulated Cockpit Motion						
Simulated Visual Reference						
Tactile Cues						
Visual Cues						

[redacted]